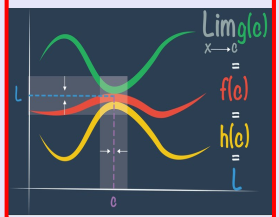
Math 261

Spring 2023

Lecture 12



Feb 19-8:47 AM

Class QZ 2:

For any $\varepsilon>0$, find $0<\delta\leq 1$ such that

$$\lim_{x\to 2}(x^2-3x)=-2$$

1) verify the limit $\Rightarrow \lim_{x\to 2}(x^2-3x)=2^2-3(2)=-2$ ✓

2) $f(x)=x^2-3x$, $a=2$, $L=-2$

3) $|f(x)-L|<\varepsilon$ whenever $|x-a|<\delta$

$|x^2-3x+2|<\varepsilon$ $\quad$ $|x-2|<\delta$

$|(x-1)(x-2)|<\varepsilon$

$|x-1|\,|x-2|<\varepsilon$

If $|x-1|<C$, then $|x-2|<\frac{\varepsilon}{C}$ $\quad$ Pick $\delta=\frac{\varepsilon}{C}$

Since $0<\delta\leq 1 \Rightarrow$ $|x-2|<1$

$-1<x-2<1$

$1<x<3$

$-2<0<x-1<2$

$-2<x-1<2 \Rightarrow |x-1|<2$

$C=2$

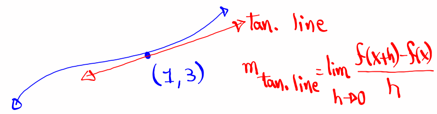
So

$$\boxed{\text{Pick } \delta=\min\left\{1,\frac{\varepsilon}{2}\right\}}$$

Feb 23-9:40 AM

Find eqn of the tangent line at $x=1$ to the graph of $f(x)=x^2-2x+4$.

at $x=1 \to f(1)=1^2-2(1)+4=3$



$$m_{\text{tan. line}}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$m_{\text{tan. line}}=\lim_{h\to 0}\frac{(x+h)^2-2(x+h)\cancel{+4}-x^2+2x\cancel{-4}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2\cancel{-2x}-2h\cancel{-x^2}\cancel{+2x}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(2x+h-2)}{\cancel{h}}=\lim_{h\to 0}(2x+\overset{\to 0}{h}-2)=2x-2$$

For $x=1$

$m=2(1)-2=0$

$y-y_1=m(x-x_1)$

$y-3=0(x-1)$

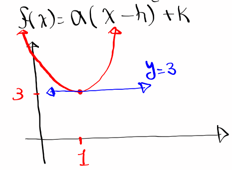
$\boxed{y=3}$

$f(x)=x^2-2x+4$

$f(x)=x^2-2x+1+3$

$f(x)=(x-1)^2+3$

$f(x)=a(x-h)^2+k$



Feb 27-8:53 AM

For any function $f(x)$, the derivative of $f(x)$, $f'(x)$ "f- Prime of x" is given by

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$ if limit exists.

Find $f'(x)$ for $f(x)=x^3-6$ using the def. of derivative.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{(x+h)^3\cancel{-6}-x^3\cancel{+6}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{x^3}+3x^2h+3xh^2+h^3\cancel{-x^3}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}}=\lim_{h\to 0}(3x^2+\overset{\to 0}{3xh}+\overset{\to 0}{h^2})$$

$$=\boxed{3x^2}$$

Think of $f'(x)$ as a formula for slope of the tan. line at any point on the graph of $f(x)$.

Feb 27-9:03 AM

Given $f(x) = 2x^2 + 4x$

1) Find $f(2) = 2(2)^2 + 4(2) = \boxed{16}$

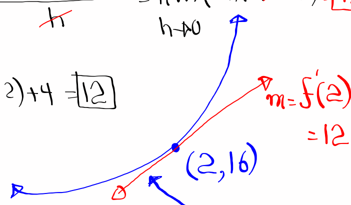
2) Find $f'(x)$ using the def. of derivative.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{2(x+h)^2 + 4(x+h) - 2x^2 - 4x}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 + \cancel{4x} + 4h - \cancel{2x^2} - \cancel{4x}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(4x + 2h + 4)}{\cancel{h}} = \lim_{h\to 0} (4x + 2h + 4) = \boxed{4x+4}$$

Find $f'(2) = 4(2) + 4 = \boxed{12}$



$y - y_1 = m(x - x_1)$

$y - 16 = 12(x - 2) \Rightarrow y = 12x - 24 + 16$

$\boxed{y = 12x - 8}$

Feb 27-9:12 AM

Consider $f(x) = \sqrt{x}$

1) Find $f(4) = \sqrt{4} = 2$

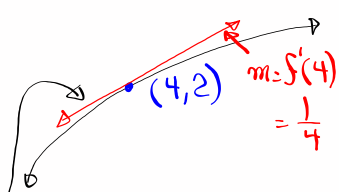
$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

2) Find $f'(x)$ using def. of derivative

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h\to 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h\to 0} \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

3) $f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$



4) Eqn of the tan. line

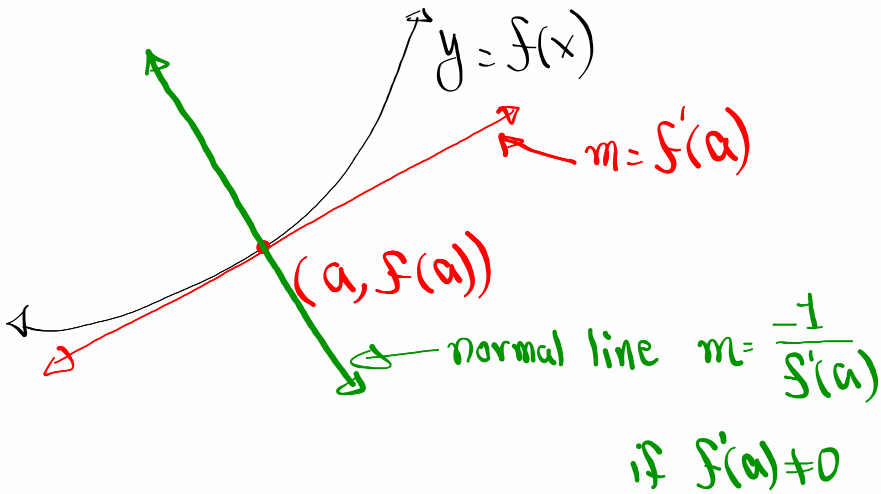
$y - y_1 = m(x - x_1)$

$y - 2 = \frac{1}{4}(x - 4) \Rightarrow \boxed{y = \frac{1}{4}x + 1}$

Feb 27-9:20 AM

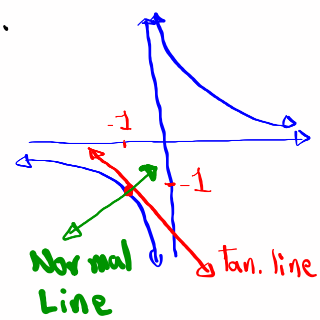
Normal line:

It is ⊥ tan. line at the tan. point.



Feb 27-9:27 AM

Find equation of the normal line to the graph of $f(x) = \frac{1}{x}$ at $x = -1$.



$f(-1) = \frac{1}{-1} = \boxed{-1}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{x - x - h}{h(x+h)\cdot x} = \lim_{h \to 0} \frac{-1}{(x+h)\cdot x} = \frac{-1}{x^2}$$

$m_{\text{tan. line at } x=-1} = f'(-1) = \frac{-1}{(-1)^2} = -1 \Rightarrow m_{\text{Norma line}} = -\left(\frac{1}{-1}\right) = 1$

$y - y_1 = m(x - x_1)$

$y - (-1) = 1(x - (-1)) \Rightarrow \boxed{y = x}$

Feb 27-9:31 AM

Other notations:

$y = f(x)$

$$\boxed{f'(x) = y' = \frac{dy}{dx} = \frac{d}{dx}[f(x)]} = Df(x) = D_x f(x)$$

$$f'(a) \to \left.\frac{dy}{dx}\right|_{x=a}$$

Feb 27-9:38 AM

Find $f'(x)$ for $f(x) = \sin x$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$\sin(A+B) = \sin A\cos B + \cos A \sin B$

$$= \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h - \sin x}{h} + \lim_{h\to 0} \frac{\cos x \sin h}{h}$$

$$= \lim_{h\to 0} \frac{\sin x(\cos h - 1)}{h} + \lim_{h\to 0} \frac{\cos x \sin h}{h}$$

$$= \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} \to 0 + \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h} \to 1$$

$$= \sin x \cdot 0 + \cos x \cdot 1$$

$$= \cos x$$

$$\boxed{\begin{array}{l} F(x) = \sin x \\ f'(x) = \cos x \end{array}}$$

Feb 27-9:41 AM